Dithering Matrix Correlation Removal for Q-weak Experiment

Peng Zang, Syracuse University

I Introduction to Dithering Error Analysis

Dithering corrections to raw asymmetries are crucial in Parity-Violating experiments, including Qweak experiment in Hall C at Jefferson Lab. Dithering corrections are composed of dithering slopes and monitor differences. Because monitor measurements are closely correlated with each other, to obtain error in dithering corrections and dithering slopes is not straightforward. Covariance calculation in error propagation is forbidding for short time scale (cycle or runlet). Here by defining uncorrelated monitors, the correlations in dithering coefficients and monitor differences are reduced, and the error in dithering corrections and dithering slopes can be calculated directly in small time scales.

$$\sum \frac{\partial M}{\partial C}\frac{\partial D}{\partial M} = \frac{\partial D}{\partial C}$$

①←The function set to solve for dithering slopes. Monitor coefficients and detector coefficients form the "dithering matrix" which show up in the formula for dithering slopes.

$$\frac{\partial D}{\partial M} = f1\left(\frac{\partial D}{\partial C}, \frac{\partial M}{\partial C}\right) \Rightarrow err\left(\frac{\partial D}{\partial M}\right) = f2\left(err\left(\frac{\partial D}{\partial C}\right), err\left(\frac{\partial M}{\partial C}\right)\right)$$

②↑Dithering slope formula, a function including all detector and monitor coefficients.

③↑Error in dithering slopes, which is a function of all errors in detector and monitor dithering coefficients.

$$\Delta_D = \sum \frac{\partial D}{\partial M}\Delta_M \Rightarrow err(\Delta_D) = f3\left(err\left(\frac{\partial D}{\partial M}\right), err(\Delta_M)\right)$$

④↑Dithering correction formula, a function of all detector slopes and monitor differences.

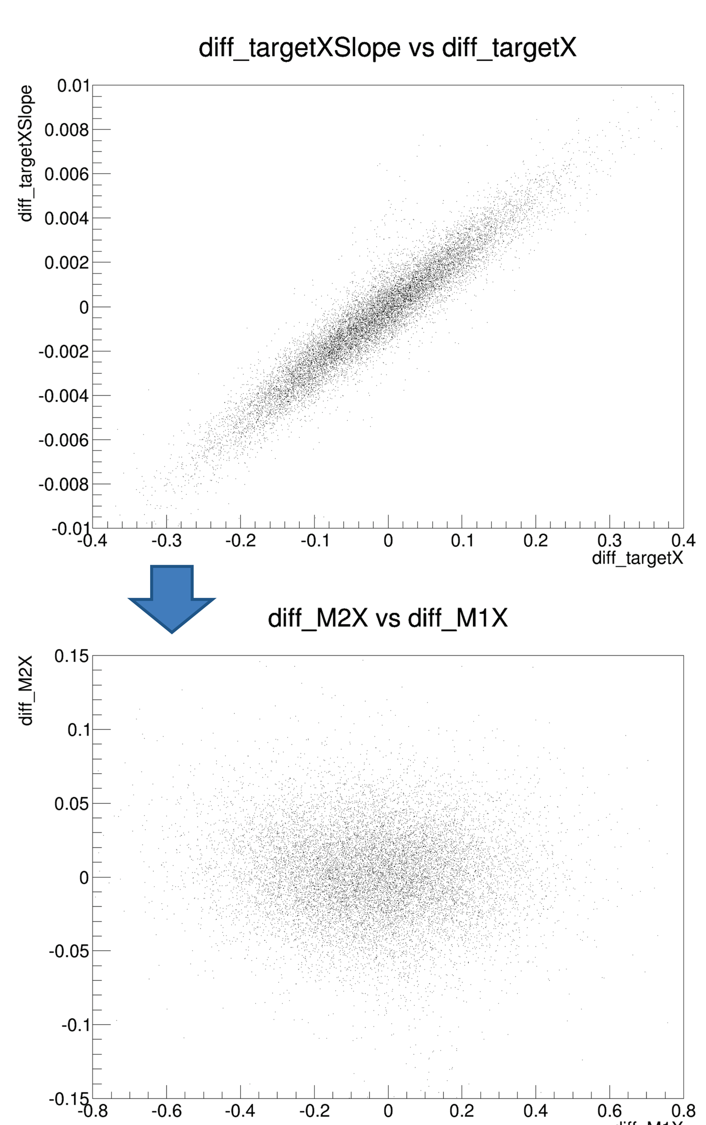
⑤↑Error in dithering corrections, including errors coming from all dithering slopes and monitor differences.

II Dithering Coefficients Correlation Removal

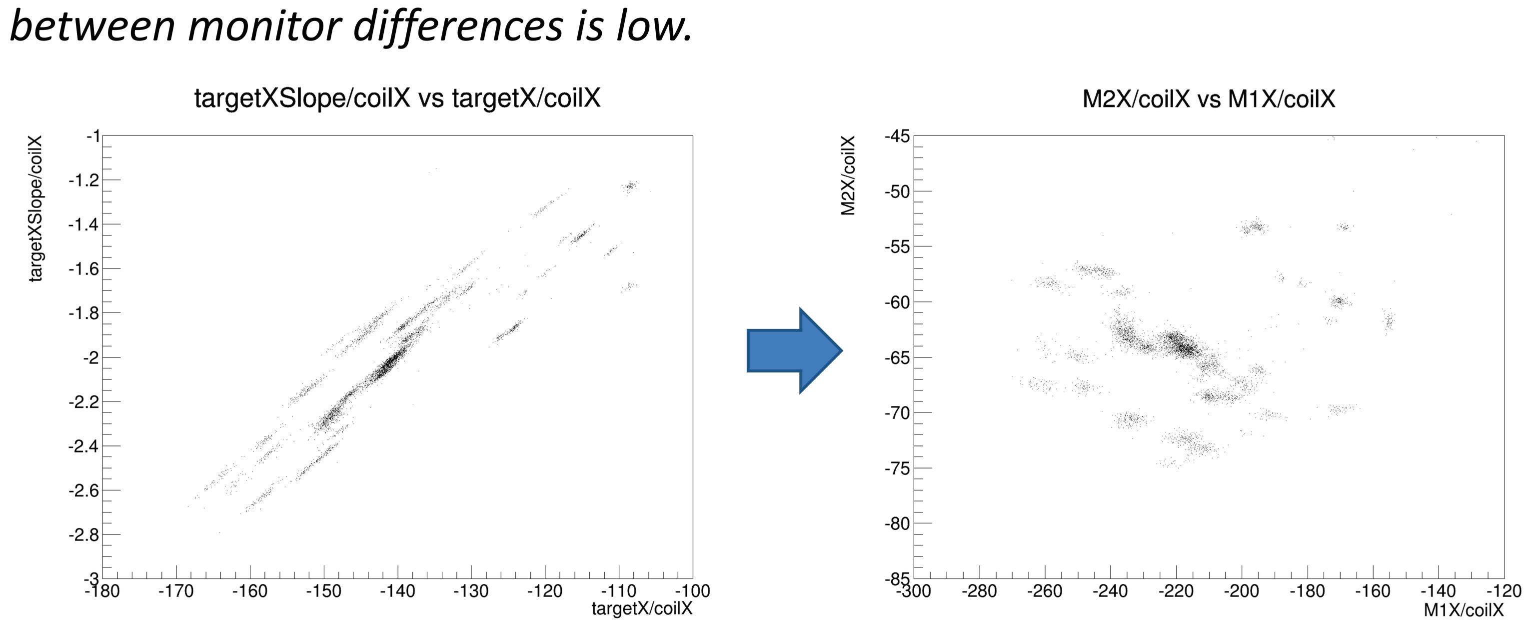
By defining a new set of monitor variables to remove the correlation between monitor differences, the correlation between monitor dithering coefficients are also removed.

$$M1 = c_{11} * target + c_{12} * targetSlope$$
$$M2 = c_{21} * target + c_{22} * targetSlope$$

①←The definition of new monitor variables, built with target variables. The constants c11, c12, c21, c22 are chosen so that the correlation between monitor differences are removed.



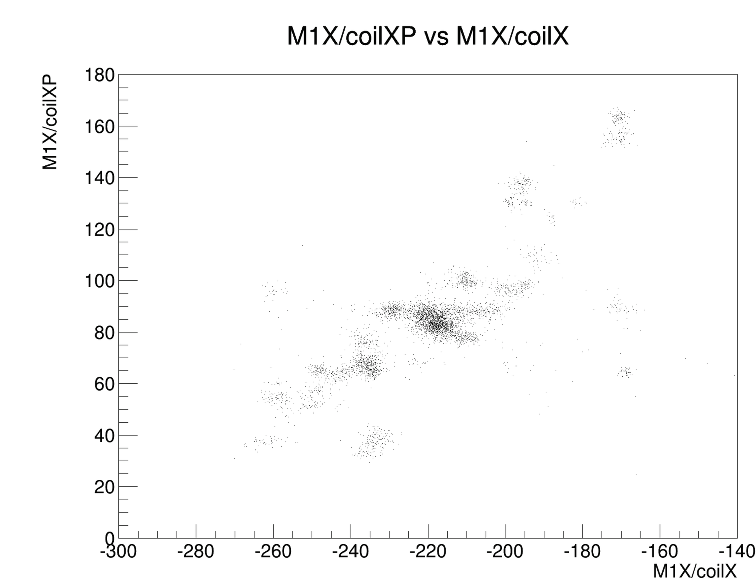
②←Monitor differences correlation at runlet level. For original monitors (top), targetXSlope versus targetX, the correlation between monitor differences is high. For redefined monitors (bottom), M2X versus M1X, the correlation between monitor differences is low.



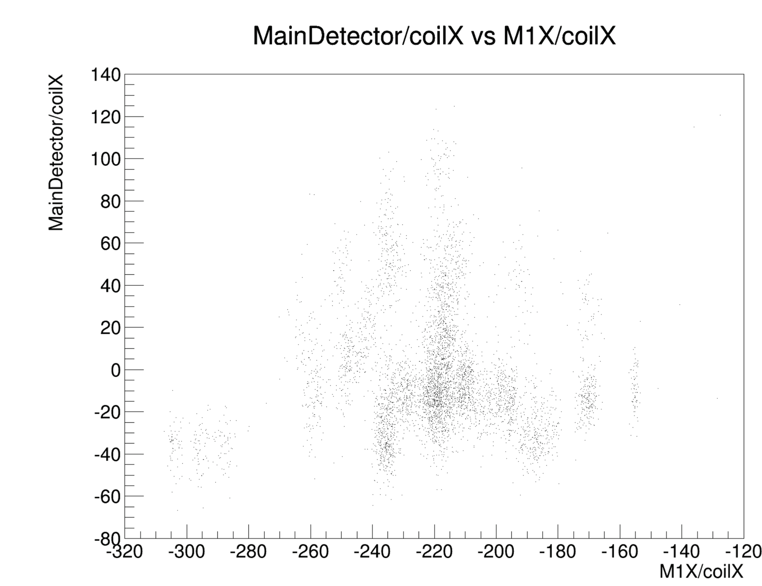
③↑Monitor coefficients correlation at cycle level. For original monitors (left), targetXSlope/coilX versus targetX/coilX, the correlation between monitor coefficients is high. For redefined monitors (right), M2X/coilX versus M1X/coilX, the correlation between monitor coefficients is low.

III Correlation-Free Matrix Double Check

In the dithering matrix, the correlation between the monitor coefficients for different coils still needs to be checked. And detector coefficients and monitor coefficients need to be correlation-free as well.



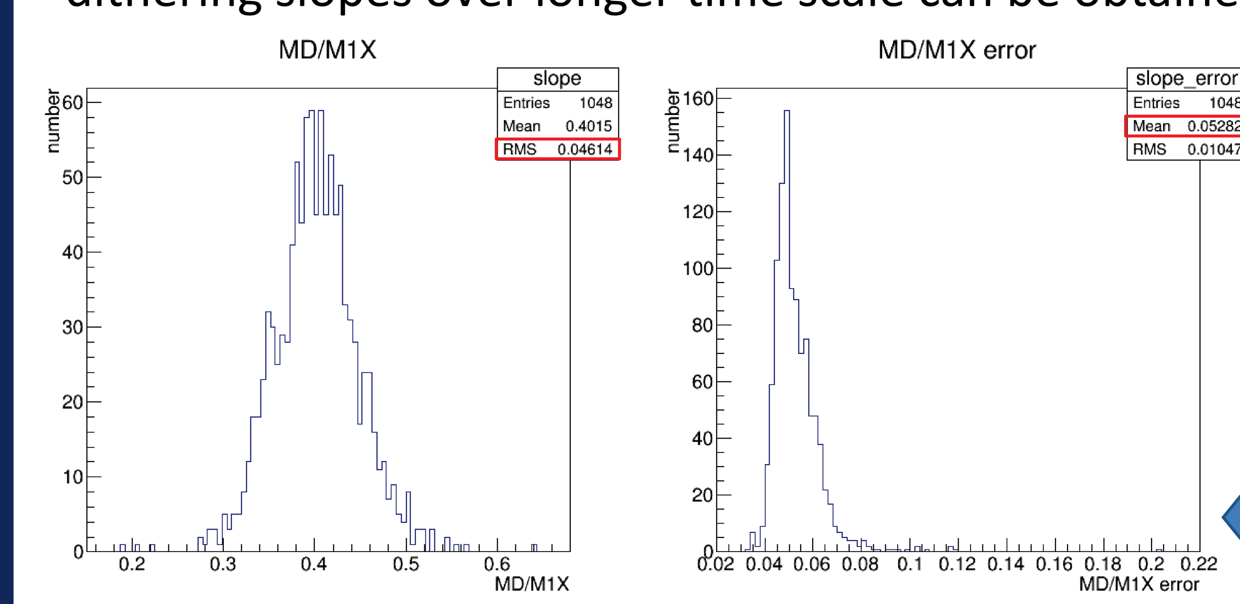
①←Redefined monitor coefficients correlation at cycle level for different coils. M1X/coilX versus M1X/coilXP. The correlation between monitor coefficients is low.



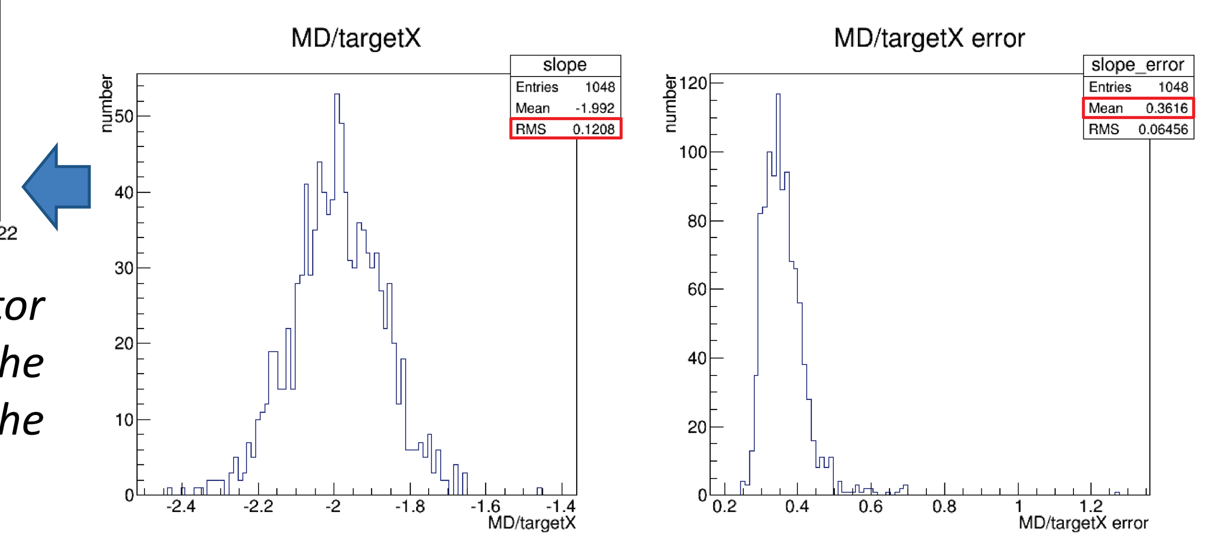
②←Correlation between detector coefficients and redefined monitor coefficients. Main Detector/coilX versus M1X/coilX. The correlation between them is low.

IV Application: Error in Dithering Slopes

With correlation-free matrix, the error in dithering slopes can be calculated via standard error propagation procedure at the smallest scale possible – cycle level. With errorbar equipped at cycle level, more reliable dithering slopes over longer time scale can be obtained.



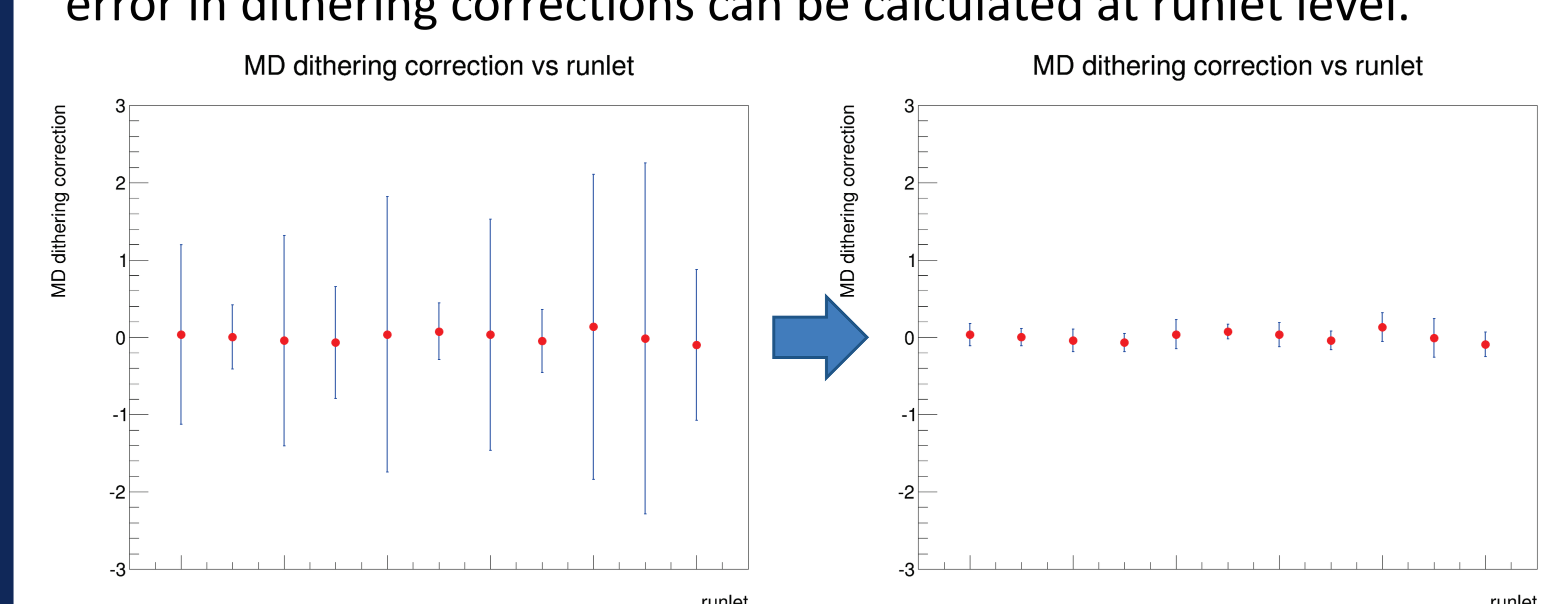
①←With uncorrelated monitors, an example of dithering slope and slope error histograms are shown on the left, including 1048 cycles. The RMS in the slope (0.046) and the mean value in the slope error (0.053) agree very well for redefined monitors.



②→With original monitor variables, main detector dithering slope and slope error histograms are shown on the right. The RMS in the slope (0.12) and the mean value in the slope error (0.36) don't agree with each other.

V Application: Error in Dithering Corrections

With error in dithering slopes obtained and the correlation between different monitors removed, the error in dithering corrections can be calculated at runlet level.



←For a single run which has 11 runlets, errorbars on the dithering corrections are calculated via error propagation procedure. With original target variables (left), the errorbars on the dithering correction are overestimated. With uncorrelated redefined variables (right), reasonable errorbars are obtained.

In conclusion, the redefined monitors reduce the correlation in dithering coefficients and monitors differences and provide a direct way to calculate error in both dithering slope and dithering correction.